NOTIZEN

A Model for Spin-lattice Relaxation in Metals

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The electromagnetic fields of the ions moving in a sound wave give rise to an interaction of electron spins with transverse phonons. The lifetime of a spin is calculated in lowest perturbational order.

If a metal is brought into a magnetic field a Pauli-paramagnetism results from the flipping of some spins over into the direction of the field. The question as to what makes the spins flip does not seem to have found much consideration so far. In this note we propose a mechanism provided by the interaction with tranvserse phonons.

If we treat a metal as the combination of a positively charged elastic solid and a negatively charged Fermi gas we obtain the following picture of a sound wave: If a longitudinal wave moves through the positive solid and the electrons stay at rest, space-charges are created which lead to an electrical field which sets the electrons into motion. A simple calculation shows that in fact the electrons move almost in the same way as the positive solid. A transversal wave in the solid would not lead to space-charges but to a current varying periodically in space and time. This current produces a magnetic field and this in turn an electrical field which tries to drag the electrons along. Again a more detailed calculation (which can be found in any treatise of absorption of ultrasound in metals)[1] shows that the electrons again move almost in step with the solid. A transverse sound wave is therefore accompanied by a periodical magnetic field whose direction is perpendicular to the wave vector and the polarisation of the sound wave. If an electron moves through this wave, it sees a periodically changing magnetic field which can change the direction of the spin.

The foregoing assumption once accepted one arrives in a straightforward way at a spin-lattice interaction model. In order that the electrons follow the ionic motion, as assumed, the ions have to exert some electric force on the electrons described by a field $\boldsymbol{E}_{\text{drag}}$. In the case of long phonon wavelength the static contribution to the conductivity cancels and this field freely accelerates the electrons (charge e, mass m, density n_0)

$$\dot{\boldsymbol{g}}_{\text{e}} = (n_0 e^2/m)\, \boldsymbol{E}_{\text{drag}} \,. \tag{1}$$

By our assumption the electronic current $\boldsymbol{g}_{\text{e}}$ is equal in magnitude and opposite in sign to the ionic current $\boldsymbol{g}_{\text{J}} = -e\, n_0\, \boldsymbol{u}$ at every moment. We get

$$\boldsymbol{E}_{\text{drag}} = (m/e)\, \ddot{\boldsymbol{u}} \tag{2}$$

with $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{r}, t)$ the ionic displacement field. Because of transverse phonons the electric field is transverse, too, and Faraday's law leads to a time variation of magnetic induction $\boldsymbol{B}$

$$\dot{\boldsymbol{B}} = -\frac{m}{e} \operatorname{curl} \ddot{\boldsymbol{u}} \tag{3}$$

and finally to a magnetic energy of one electron,

$$h_{\text{int}} = -\mu_{\text{B}} (\boldsymbol{\sigma} \boldsymbol{B}) = (m\, \mu_{\text{B}}/e)\, (\boldsymbol{\sigma} \operatorname{curl} \dot{\boldsymbol{u}}) \tag{4}$$

with Pauli matrices $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and Bohr magneton μ_{B}. Equation (4) is an interaction hamiltonian for the scattering of electron spins by transverse phonons.

We introduce phonon creation and annihilation operators $a_{q\lambda}^{(+)}$ with wave vector $\boldsymbol{q}$, polarization $\boldsymbol{e}_{q\lambda}$ and frequency $\omega_{q\lambda}$

$$\boldsymbol{B} = \sum_{q\lambda} \boldsymbol{b}_{q\lambda} \left(a_{q\lambda}^{+}\, e^{-i(\boldsymbol{q}\boldsymbol{r})} + a_{q\lambda}\, e^{i(\boldsymbol{q}\boldsymbol{r})}\right) \tag{5}$$

where

$$\boldsymbol{b}_{q\lambda} \equiv \left(\frac{m^2 \hbar\, \omega_{q\lambda}}{2\, e^2 \varrho\, \Omega}\right)^{1/2} [\boldsymbol{e}_{q\lambda} \times \boldsymbol{q}]$$

with volume Ω and ionic mass density ϱ.

To get an impression of the order of magnitude we estimate $\langle \boldsymbol{B}^2 \rangle$ averaged over an ensemble of free phonons.

$$\langle \boldsymbol{B}^2 \rangle = \sum_{q\lambda} \frac{m^2 \hbar\, \omega_{q\lambda}}{2\, e^2 M\, n_0\, \Omega}\, q^2 \operatorname{ctanh}\left(\tfrac{1}{2} \beta \hbar\, \omega_{q\lambda}\right) \tag{6}$$

λ, transverse polarization.

This leads for temperatures small compared with the Debye temperature Θ to

$$\sqrt{\langle \boldsymbol{B}^2 \rangle} = \frac{m\, \pi}{M\, e} (3\, \varrho\, c_{\text{s}}\, \hbar)^{1/2} \left(\sim 15\, \frac{\text{Vs}}{\text{m}^2} \;\; \text{for Cu} \right)$$

with ionic mass M and velocity of transverse sound c_{s}.

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In the form of second quantization with two-component Fermi fields $\psi_k^{(+)}$ the interaction hamiltonian, Eq. (4), reads in the case of an electron gas

$$H_{\text{int}} = -\mu_B \sum_{kq\lambda} [\psi_k^+ (\boldsymbol{\sigma}\, \mathbf{b}_{q\lambda})\, \psi_{k+q}\, a_{q\lambda}^+ + \psi_k^+ (\boldsymbol{\sigma}\, \mathbf{b}_{q\lambda})\, \psi_{k-q}\, a_{q\lambda}] \,. \tag{7}$$

The lifetime $\tau_k(\omega)$ of a particle with fixed spin, energy and momentum $\boldsymbol{k}$ is calculated in lowest order of the coupling constant and leads to a result differing from usual electron-phonon coupling by its order of magnitude and by the higher power of transferred momentum arising in the matrix elements $\mathbf{b}_{q\lambda}$. Without external magnetic field the lifetime is independent of spin and we get

$$\frac{1}{\tau_k(\omega)} = \frac{\pi\, \mu_B^2}{\hbar^2} \sum_{q\lambda} \mathbf{b}_{q\lambda}^2 \Big\{ [f_B(\omega_{q\lambda}) + f_F(\omega_{q\lambda} + \omega)] \times \delta\left(\omega - \frac{\varepsilon_{k+q}}{\hbar} + \omega_{q\lambda}\right) + [f_B(\omega_{q\lambda}) + f_F(\omega_{q\lambda} - \omega)] \times \delta\left(\omega - \frac{\varepsilon_{k+q}}{\hbar} - \omega_{q\lambda}\right) \Big\} \,. \tag{8}$$

The energy of free electrons is denoted by ε_k and $f_B(\omega)\,[f_F(\omega)]$ stands for the Bose-(Fermi-)distribution. The evaluation of the integral in Eq. (8) is carried out with the standard approximations and leads in the zero temperature limit to

$$\frac{1}{\tau_k(\omega)} = \frac{3\,\pi\, m\, q_D}{40\, M\, k_F} \left(\frac{\hbar\, \omega}{k\, \Theta}\right)^4 |\omega| \,, \quad \hbar\, |\omega| \ll k\, \Theta \,, \tag{9}$$

q_D = Debye cut-off = $(6\,\pi^2\, \varrho/M)^{1/3}$,
k_F = Fermi momentum.

For $\hbar\,|\omega|$ comparable with or greater than the Debye energy $k\,\Theta$, the lifetime becomes constant, approximately equal to its value at $\hbar\,|\omega| = k\,\Theta$ from Equation (9).

Contrary to the electron-phonon interaction, which results in $\tau^{-1} \sim |\omega|^3$, we find here a dependence on the fifth power in $|\omega|$ for small energies. The maximum damping rate, i. e. for $\hbar|\omega| \gtrsim k\,\Theta$, is smaller by a factor $\sim 10^{-5}$ than in the electron-phonon case.

Considering the spin-lattice relaxation, i. e. the lifetime of a single electron spin in a surrounding medium, we have to ask for the transition probability, $[\tau_{sk}(\omega)]^{-1}$, of scattering the initial state into final states with spin reversed. This quantity differs from the total transition probability calculated via Equation (8). Projecting onto intermediate states with spin reversed only, we get from Eq. (8)

$$\frac{1}{\tau_{sk}(\omega)} = \frac{3\,\pi\, m\, q_D}{160\, M\, k_F} \left(\frac{\hbar\, \omega}{k\, \Theta}\right)^4 |\omega|\, [3 - (\cos \varphi_{sk})^2] \tag{10}$$

for the spin relaxation rate at zero temperature. In the case of transverse spinpolarized electrons its magnitude is equal to 3/4 of the total damping rate. The quantity φ_{sk} denotes the angle between spin and momentum of the incident electron. Electrons with longitudinal spinpolarization, $\varphi_{sk} = 0$, experience only 2/3 of the transverse damping rate. This result may be understood classically because the spin scattering consists in a spin precession around the axis of magnetic field which points normal to momentum and polarization of the special interacting phonon. As the essential constribution to the $\boldsymbol{q}$-summation, Eq. (8), arises from a small azimut angle with respect to a plane normal to the electron momentum $\boldsymbol{k}$, one of the two transverse phonons is polarized normal to $\boldsymbol{k}$. If the incident spin is aligned with $\boldsymbol{k}$ this mode does not contribute to the spin precession, because $[\boldsymbol{e}_{q\lambda} \times \boldsymbol{q}]$ is parallel with $\boldsymbol{k}$. Whereas if the incident spin points normal to $\boldsymbol{k}$ both modes contribute. The lower limit for the relaxation time resulting from Eq. (10) is about 10^{-9} sec for a monovalent metal.

[1] C. Kittel, Quantum Theory of Solids, Wiley & Sons, New York 1963.
A. B. Pippard, The Dynamics of Conduction Electrons, Gordon and Breach, New York 1965.